### An Analysis of Persian Lilipoush Vaults: Mastery and Creativity of Traditional Architects in Construction Method and Final Form

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### Abstract

Barrel vaults are one of the component elements of curved roofs in Iran. They play a significant role in creating geometric forms and spatial quality. Among a variety of barrel vaults which are most widely used, annular ones have different construction methods. These annular vaults have various types of brick bonding. Among them, the brick bonding of Lilipoush vaults is very different. This paper strongly believes that the buttress size and height of vaults, the availability of materials, and the mastery and creativity of traditional architects were all effective on the construction technology and brick bonding of Lilipoush vaults. This paper tried to identify the geometry of annular vaults and the type of brick bonding used in Lilipoush vaults. The fundamental question of this paper is that "How effective were the mastery and creativity of traditional architects at the construction method and final form of Lilipoush vaults?" To answer this question, this paper adopted an applied research methodology with deductive reasoning. Data were collected based on library and field studies, observation and semi-structured interviews with traditional architects. The results indicated that the special brick bonding of Lilipoush vaults was an outstanding achievement in building annular vaults. In fact, the limited availability of materials and the mastery of traditional architects eliminated the need for wood supports and counterforts in this type of vaulting. This can be partly because of the dry climate of the central villages of Iran which had limited resources of timber. To overcome this deficiency in materials, traditional architects used these vaults to cover open spaces in rural areas.

Keywords: building method, annular barrel vaults, Lilipoush vault, traditional architects.

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### Introduction

The dry climate of Iran and the limited resources of timer have led the Persian architects to find new solutions for closing spans and covering open spaces. As a result, a wide variety of tiercerons, vaults, and domes were invented for roofing open spaces with different dimensions (Godard et al. 2005). Among others, the semicircular structure of vaults made them suitable for covering open spaces and also decorating entrances. The rich diversity of Iranian vaults in terms of force behaviors and forms requires an independent study (Memarian 2012: 89). One of the roofing types used in old buildings is the annular vault, which can be constructed without timbers. An annular vault is formed by the extrusion of a single curve parallel with the imposts which continues in the entire roofing or in a part of it. Moreover, the type of brick bonding and mortar used for vaults play a key role in the resistance of vaults (Valibeig 2012: 2). One of the methods used in the annular vaults is the *Lilipoush* method. This method was generally used in rural areas and hot and dry climatic zones. The brick bonding of Lilipoush vaults, which were usually constructed of stone, adobe or brick, took many different forms over time. The construction technology resulted from the geometric form, the capability of architects and the accessibility of materials were all contributive to those changes.

Traditional architects have used vaults and domes to cover spaces in the hot and dry areas of Iran for many years. Domes were mostly used for public places and in urban monuments, but vaults were only used to cover a space with a roof. In different eras. traditional architects have built different types of vaults each having specific methods, unique characteristics, and different materials. Lilipoush vaults were mostly used in rural areas because of the lack of materials and the absence of experienced architects in such areas. They were resulted from the artistic creativity and the experience of Iranian architects in building vaults over a long period of time.

A better understanding of the construction methods and the type of brick bonding used

in various vaults in Iran will offer good solutions for the preservation and restoration of these architectural elements. It will also save the techniques of the experienced architects from being forgotten -architects who adapted different forms to be suitable for different climates with access to primitive tools and limited materials like soil. The present paper attempted to investigate the importance of geometry, the type of brick bonding, and the creativity of traditional architects in building Iranian brick vaults (Lilipoush vaults). It is important because the mastery and technical heritage of past architects may help such vaults to be restored and revitalized especially in rural areas. To achieve this goal, this paper tried to answer the following questions:

- With regard to the form of the spatial plan, how did traditional architects create different Lilipoush vaults to cover spaces with roofs?

- In the process of constructing this type of vault, how was it possible to cover a rectangular plan without molding?

- With regard to the diversity of Iranian vaults, what was the reason of using Lilipoush vaults in rural areas rather than other types?

### Background

Most of the researchers who studied the relevant issues of vaults were only concerned with their shape and construction method (Besenval, 1984) (Kashani, 1987; Memarian, 2007; Prinia, 1994, Zomorshidi, 1994). Some writers (including Godard, 1936; Besenval, 1984) investigated the structure of Iranian vaults. Some others analyzed the manner of resistance in them (South, 2005). There are only few studies that explored the component elements of these vaults (Memarian, 2012; Zomorshidi, 1994; Tehrani, 2012). The present paper has tried to explain the manner of emerging structural elements along with geometric forms and construction technologies. Therefore, it is the first attempt to analyze the qualities of Lilipoush vaults in roofing with regard to the abilities of local architects. Using precise pictures, conducting careful filed studies, and benefiting from the opinions of experienced local architects, this paper provides the details of construction method and the manner of fixing different

elements of Lilipoush vaults. **Theoretical Framework** 

With regard to the objectives and the nature of this study (which is multi-variable), and due to the incomprehensiveness of theories (the fact that theories could not respond to the multiple variables affecting this article), a combination of these theories were used. In fact, this paper used the theories that were more compatible with the multiple variables (the geometry of plan and the ability of traditional architects) it was concerned with.

Construction forms have two characteristics: Physical one (materials and construction methods) and geometric one (the geometric form). This second characteristic consist of two parts: theoretical dimension (geometry practical on paper) and dimension (application of geometry in practice). They are both concerned with the coordination they have with each other and the relationship of geometry with different factors like shapes, angles, dimensions and properties of the said items. In the practical dimension, which was performed by the traditional architects, the manner of creating the shapes with available materials and the close relationship of geometry with its application is the focus of attention. Traditional architects took different theoretical and practical steps to create their art. They did their job with the help of applied sciences, special technics, materials available at that time, environmental factors, mastery over different sciences, what they have learned from their predecessors, and their own skills and experiences. Vaults have been constructed from different materials, like adobe and brick, and have taken a variety of shapes over time. This development in shape and materials was influenced by theoretical-practical geometry (Golombek, 1988) and the architect's abilities.

### **Research Methodology**

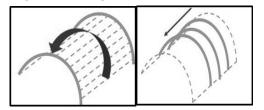
To answer the questions of this paper and to prove the hypothesis, a retrospective structure was used. The type of research is an applied one, because in addition to explaining the brick bonding of Lilipoush vaults and the abilities and creativities of traditional architects, it tries to suggest a number of solutions for revitalization of this type of roof in rural areas, especially in villages deprived

from modern technologies. Variables of the research are qualitative. This research adopted a deductive methodology: it first studied Lilipoush vaults and then analyzed its subcategories (Eshaghian 2016: 32). Data were collected based on library and field studies, observation and semi-structured interviews with traditional architects; and these data were used to analyze the structure of different annular vaults in terms of geometry and construction. We started the research by collecting previous studies about the shape and construction of different types of barrel vaults, and then we did fieldwork to analyze them. After that. based on information gained from interviews with architects, we reproduced the image of vaults in Auto CAD. At the end, we used the software to simulate the type of brick bonds used in the vaults, and once again, we showed them to architects of other regions for consultation.

### **Construction and Tools of Barrel Vaults**

A barrel vault is a vault that is can be constructed with different materials, brick bonds and plans. Some of the old examples of these vaults can be found in Haft Tappeh, underground tombs of Choghazanbil, and Tappeh Noushin Jan in Malayer. These semicircular structures are formed by the extrusion of a single curve on two parrel walls, which have equal heights (Memarian 2012: 136).

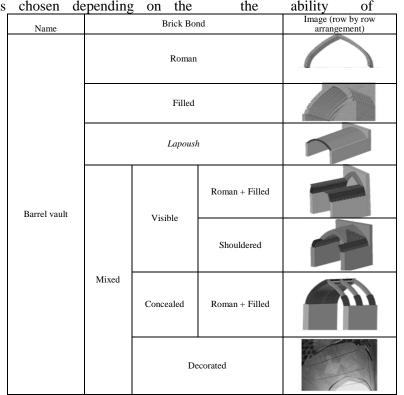
To build a barrel vault, the first step is to construct two parallel walls perpendicular to the ground, on which plaster tiercerons are laid. When tiercerons are drawn imaginary on the imposts of the parallel walls, we can have a semicircular shape. This semicircular shape is called a barrel vault, a tunnel vault or a wagon vault (Figure 1-2).



F 1. A barrel vault with circular motion (Source: Negarandegan) F 2. A barrel vault with horizontal motion (Source: Negarandegan)

This is done with different construction methods from materials such as brick and stone (Valibeig et al. 2012).

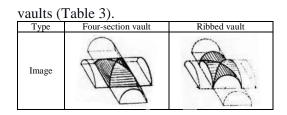
There are five types of brick bonding used in barrel vaults (Table 1). The type of brick bonding was chosen depending on the material of the vault, mortar of the vault, dimensions of the space that the vault covered, visibility or concealment of the main structure of the vault, construction of the vault on a sloping or level surface, and the ability of the architect.



T 1: Different types of brick bonding .

#### Theoretical geometry and the structure of vaults (introduction of different types of barrel vaults)

In Iran, vaults are categorized according to functions and climate. However, there is also a need for a better classification due to the rich diversity of Iranian vaults concerning force behaviors and forms. The theoretical geometry of a barrel vault is explained as the extrusion of a single curve forming a semicircular shape. It was used as the primary model of many other vaults including double barrel vaults and ribbed vaults, because it needed simple construction method and basic technologies. Four-section and four-ribbed vaults are formed when two half cylinders are combined perpendicular to each other (Table 2). Kermanipoush and Lilipoush vaults are also considered as barrel



T 2: Theoretical geometry of forming double barrel vaults and ribbed barrel vaults.

### Construction method of barrel vaults (practical geometry)

Brick bonding of a barrel vault is classified according to the conditions of construction, existence or non-existence of molding, timber accessibility, and the geometrical conditions of the plan, into five types: Roman, filled, Lapoush, mixed and decorative (Table 1). In

terms of construction method and technology, barrel vaults are of three types. In the construction method that do not use molding, the first course of bricks is laid diagonally on the entablature. This clever method obviates the need for molding and utilizes buttresses (Karouchi, 2004: 35) (Table 4).

(Isaiot	iem,	2004: 5	5)(14	JIC <del>4</del> ).		
Тур	e	Specific ations	Image		Structural arrangeme nt	
Inclin ed barrel vault Thr ee- wal 1		Constru cted on two walls	AN			
		Constru cted on three walls				
Kerman h vai		Generall y construc ted on four walls			F	
Lilipoush vault		Unilater al	Const ructed on three walls			
		Bilateral	Const ructed on four walls	120		
Four-se barrel		Constru cted on four imposts	X			
Four-ribbed barrel vault		Constru cted on four walls theoretic al and practical geometr y similar to barrel vault				

#### T3. Different types of roofing structures derived from barrel vaults (source: Negarandegan).

Type of vault	Construction method	Specifications of construction		
Barrel vault	With molding	Molding acts as bearer. Materials are wood, metal or plaster		
	Without molding	Buttress is used		
		No buttress is used (Lilipoush vault is used)		

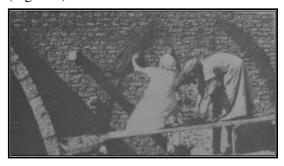
### T4. Different construction methods of barrel vaults (Negarandegan).

In different parts of Iran, one of the below methods were adopted for construction depending on the local conditions:

1. A barrel vault may be constructed with molding. But this method was rarely used, because it was possible to eliminate molding. Moreover, in some areas, it was difficult to prepare a timber molding. Even if it was easy, the quality of wood was at risk of termite invasion. In this method, tiercerons are parallel with each other, and the last wall is stretched up to the end of the vault and continues. The height of the wall in this method is equal with the height of the vault, and the roman brick bonding is used.

2. In the second method, when the wall is finished and the base of vault is ready, the last wall is constructed in the same height of the ceiling, and the curve is drawn above it. After that, the brickwork of each course is laid on the previous one. The construction method is rough brick arch, and this type of ceiling is called tunnel vault (Farzaneh<sup>1</sup>, personal interviews with authors, 2011). This method was widespread in Egypt (Fathi, 2003: 46-49). According to the master architects, the construction method is as follows:

-This vault is formed on four continuous walls which are constructed simultaneously and transfer the load of vault to the ground (Farzaneh, personal interviews with authors,  $2011)^2$ . If there is no wall at the back of the structure, a temporary partition would be used which only acts to designate and draw the shape of curving (Mohsen Naeini<sup>2</sup>, personal interviews with authors, 2011) (Figure 3).

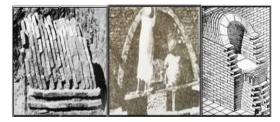


F 3. Second step, forming the curve on the wall using mortar. Place of construction: Egypt (Source: Besenval 2000, Fahi, 2003)

Architects used plaster to have a circular shape on the back wall by means of rope and nail. The height of the arch depended on the space it was covering and the accessible materials. In the process of construction, usually three ropes were used to prevent bricks from being jutted and to level the interior surface of the vault (Farzaneh, personal interviews with authors, 2011) (Figures 4-6).



F 4. Third step, forming the curve on the curved line of wall on the back wall. Place of construction: Egypt (Source: Besenval 2000, Fahi, 2003).



F5. Fourth step, inclination of brick towards wall. Place of construction: Egypt (Source: Besenval 2000, Fahi, 2003).



F6. Leveling the vault using ropes. Place of construction: Abarkouh, Yazd Province, vault restoration (Source: Negarandegan).

This method was used in the absence of molding and timber in a way that the back wall reached the primary height of the vault at the end of side walls. After that soldier bricks were laid on the last course of the wall. The vault is constructed with two angles, the first one is the angle between the vault and the back wall and the second one is the angle between the vault and the curve on which the vault is constructed. Therefore, the imaginary curve of the vault is drawn on the back wall before the construction of the vault.

In this method a course of brick is laid, and for the next course bricks are laid in a way that vertical joints are not positioned above one another on consecutive courses to ensure resistance. Then this process is repeated to have the final shape of the vault. For having an inclination towards the perpendicular line, the gap of joints are filled with crushed bricks. The angle between the bricks and the wall depends on the type of the arch but is never zero. This gap is filled with crushed brick (Farzaneh, personal interviews with authors, 2011).

3. The third method for construction of barrel vaults is the method used for Lilipoush vaults. In this method, there is no need for molding or buttresses.

## Brick bonding of Lilipoush vault in rural houses

When the wall is finished and the base of the vault is ready, the last wall is constructed in the same height of the vault's base, and from the two corners the construction of vault begins. In this method, the courses are laid on the top of each other inclining towards the center. Another technique is to begin the brick from four corners and finish it in the center -- this is called Lilipoush (Kashani 1987: 88-89). One of the oldest examples of Lilipoush vaults was constructed in Tell al-Rimah, Iraq (Figure 7). In Iran, the last remaining vaults belong to the Safavid and Qajar periods. They were usually constructed in hot and dry climates of Iran such as Yazd and Kerman Provinces, where the lack of timber encouraged people to use their own local materials. In building a vault, timber is only used for molding as a temporary structure which is removed after completion. When there is no timber, the next choice possible is to use buttresses. But when there is no timber, nor there are buttresses, vaults are constructed in Lilipoush method. In general, there are two possible methods to construct a vault: with the help of molding or with the help of a vertical buttress. The

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height of the buttress may be above or below the vault (Figure 8). However, in Lilipoush method, neither of them is needed. Therefore, there is no need for the buttress, which increases the consumption of materials and obscures the vision.



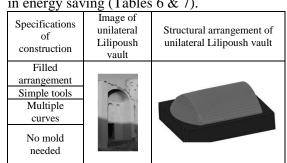
F 7. One of the oldest examples of Lilipoush vaults was constructed in Tell al-Rimah, Iraq. (Source: Besenval 2000: 108).



F 8. A barrel vault with buttress, Fire temple of Firouzabad, Fars Province (Source: Negarandegan).

Lilipoush vaults were usually used for buildings where aesthetic qualities were not important, and the only purpose was to cover a space in rural houses. The desirability of this type of roofing is partly because of its simple method of construction obviating the need for experienced architects, who were few in rural areas. Moreover, utilization of inexpensive and accessible materials like adobe adds to the desirability of Lilipoush vaults in arid regions where timber was difficult to find. It is also to be noted that the low height of this roofing provided better environmental coziness (Table 5).

To have a Lilipoush vault, three walls should be constructed simultaneously (Table 7-A). Construction of a high curve causes the back wall to be thicker (Table 6 – Plan section). To avoid the excessive thickness of this wall, there was a variety of geometrical solutions in different regions. One of them is to lower the height of the vault. The reduction of this wall in thickness added to the usable space of the house. Moreover, a lower ceiling caused the place to be smaller – thus, it was effective in energy saving (Tables 6 & 7).



### T 5. Specifications of brick bonds in Lilipoush vaults (Source: Negarandegan).

A lilipoush vault is usually built of adobe and without molding. However, in some mountainous areas they were also built of stone for which molding is needed because stones are heavy and molding will help curbing the vault. It is to be noted that building the vaults of stone was only possible in regions that was rich in timber (Figure 9).

# 5-1. Different types of brick bonding in Lilipoush vaults

In the construction method of a Lilipoush vault, four walls are constructed in four sides with equal heights. Bricks are first laid on the walls in all four corners two by two to reach to the point where the arch is to be started. From this point, laying the bricks continues like that which is used for the back wall. This method may be completed in two ways.

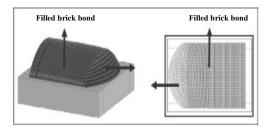
5-1-1. Unilateral Lilipoush method: In this method, only at the end of the vault, Lilipoush method is used and at the beginning, a high-edge tierceron is made using a timber molding. The high-edge tierceron is made with roman arrangement, and the gap it has with the vault is filled with bricks (Figures 10 & 11). In order to conceal the last part, different types of concealment are used to fill the gap between the vault and the high-edge tierceron. This method in architectural jargon is called unilateral inclination.

	Elevation			Plan			Perspective	
Arch Span: X, Height: H	Arch Span: X, Height: Y	Arch Span: Q, Height: Z	Arch Span: X, Height: H (Back Wall Thickness: K)	Arch Span: X, Height: Y (Back Wall Thickness: L)	Arch Span: Q, Height: Z (Back Wall Thickness: L)	Arch Span: X, Height: H	Arch Span: X, Height: Y	Arch Span: Q, Height: Z
Q	Y X	H	Q>X					

T 6. The effect of the height of arch on the thickness of the back wall (Source: Negarandegan).

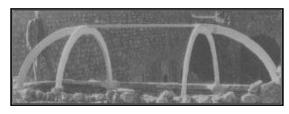
Row	А	В	С
Type of vault	Construction of vault on three walls	Increase in the thickness of the designated wall	Plan to avoid thickness of wall
Plan of bilate ral Lilip oush vault			

T 7. Construction method and plan of barrel vault on three walls (Source: Negarandegan).

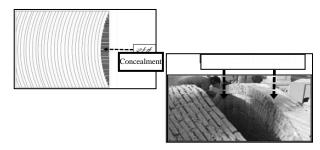


F 10. Plan and elevation of bilateral Lilipoush vault (Source: Negarandegan).

**Bilateral Lilipoush method**: The vault is constructed from both ends using Lilipoush method. Therefore, the central part of the vault is filled by concealment. The method that is usually used for completion in this stage is a method called *Gordemahi*. In this method, the vault is inclined towards two directions (Valibeig 2012: 49) (Figure 12 and Table 7). A bilateral Lilipoush vault has a better functionality than a unilateral one. Because in the unilateral type we need timber to build the high-edge tierceron and complete concealment, while in this method no timber



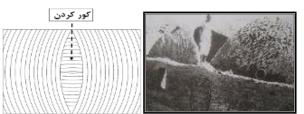
F 9. A plaster molding to construct a stonemade vault (Source: Personal archive of Dr. Fakhar Tehrani).



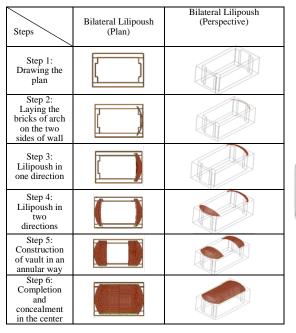
F 11. Concealment of unilateral Lilipoush vault (Source: Negarandegan, Abarkouh, Yazd Province).

### is needed.

**Elongated vault**: If in the middle of the bilateral Lilipoush method, a plaster molding is used to lay a middle tierceron above it which has its two legs on the side walls, the final vault will be an elongated one (Godard, 1936). The number of plaster molds varies and is constructible in rectangular plans. Moreover, in some buildings, an elongated vault is used in the veranda. An example of this type of vault can been seen in Ali Qapu Palace, located in Isfahan.



F 12. Plan and elevation of bilateral Lilipoush vault (Source: Besenval, 2000: 114).



T 7. Step by step construction of bilateral Lilipoush.

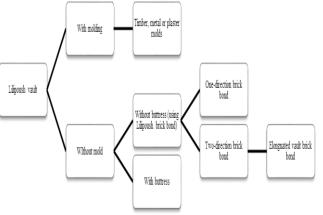
### Conclusion

Lack of timber and its damages (like termite) made it widespread in Iran to use local materials for construction of vaults. Barrel vaults are a good example of this claim. This type of vault could be constructed in a number of methods with different brick bonds such as roman, filled, Lapoush, mixed and decorative. In this method, the need for timber is in the minimum level, and it was only needed in the roman brick bonding. Filled brick bonding did not need molding or timber, and it could be completed using buttress which increased the consumption of materials. Another method for construction of barrel vaults was Lilipoush method. This





F 13. Plan and a picture of a bilateral Lilipoush vault (Source: Godard, 1963).



#### F 14. Classification of Lilipoush construction methods (Source: Negarandegan)

method was optimum among others due to the obviation of molding and buttress. In this method, a decrease in the consumption of materials and also in the arch caused the height of the lower space to be less -saving energy in the hot and dry climates (Section 5). Another advantage of this method over others was its simplicity which did not need experienced architects to be built. The creativity and experience of architects helped them to achieve a roofing method, implementation of which was even easy for inexperienced villagers. However, it is to be noted that this type of vaulting was the fruit of many years of experience and knowledge. The construction method of Lilipoush vault depends on the space it covered. These methods include bilateral, unilateral and middle tierceron methods. Benefiting from bilateral walls for rectangular spaces would add to the thickness so that the architect may lay the first course of the vault on the corners without tierceron or molding. By putting the crushed brick under the courses, gradually the vault gets away from the wall and covers the space. Here, the horizontal and vertical motion of the bricks are combined to each other to make the distance from the edge of the wall possible, and thus covers the space.

#### Endnotes

1. Farzaneh: Gholamreza Farzanei Mohammadi, known as Farzaneh, born in 1926 in Naein. He was one of the famous architects of Iran. Some of his works include restoration of Masjid Jame Ardestan, restoration of Masjed Jame Natanz, restoration of Fin Garden of Kashan, and restoration of Afooshteh historic buildings (Sajadi Naeini 2009, 183).

2. What is meant by Mr. Farzaneh Naeini is the bilateral Lilipoush vault which is laid on four walls. A barrel wall is either built on two parallel walls or built using a buttress, the central part of which is like a Lilipoush vault.

3. Mohseni: Yousef Mohseni Naeini was born in Esfahan suburbs. He was a descendent of Mohammad Hossein Mohseni Naeini who was a genius in the field of restoration in Iran (Sajadi Naeini 2009, 140).

These terms are used by traditional architects. Also, some academic professors, like Dr. Farhad Tehrani, use this term. The term is still in use in cities like Abarkouh, Yazd Province.

Acknowledgements

4. We would like to thank the late Mr. Gholamreza Farzaneh Naeini and Mr. Yousef Mohseni Naeini who generously shared their invaluable knowledge with us.

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